Correlation & Regression

correlation

→ Degree of association between two variable

→ correlation can be negative positive or zero

→ $-1 \leq r \leq 1$

→ for Direction Relation b/w x & y

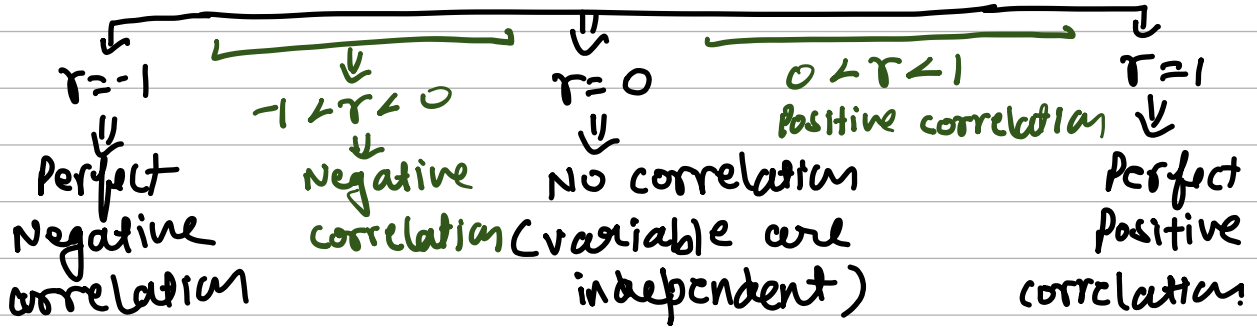
$x \uparrow y \uparrow$ or $x \downarrow y \downarrow$

r is positive

→ for inverse Relation b/w x & y

$x \downarrow y \uparrow$ or $x \uparrow y \downarrow$

r is negative

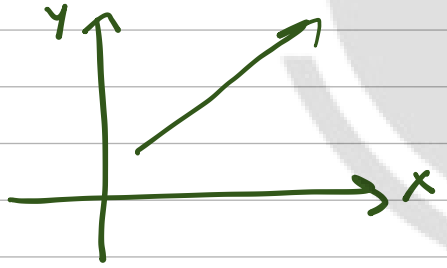


Methods of calculating correlation

- Graphical method
  - Scattered Diagram method
- non Graphical method
  - Karl Pearson method
  - Spearman method
  - concurrent Deviation

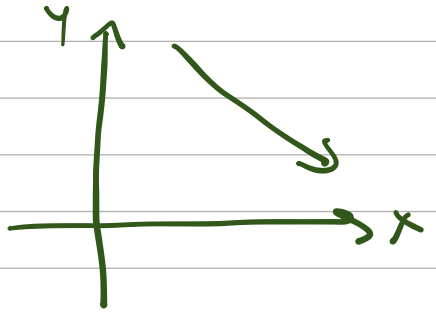
Scattered Diagram method

Plot the values of two variable on graph



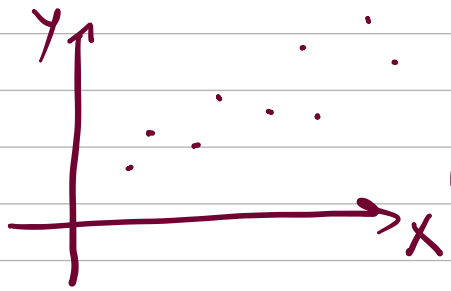
for straight line
(lower left to upper right)

$r = 1$

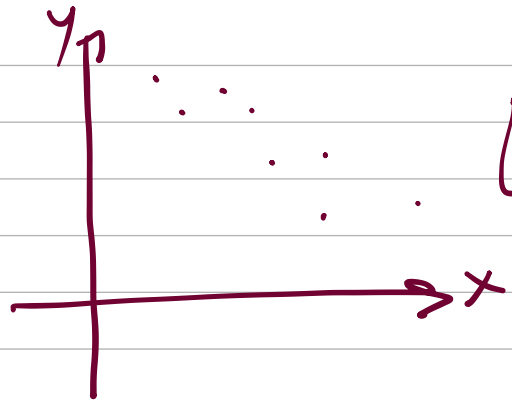


for straight line
(upper left to lower right)

$r = -1$

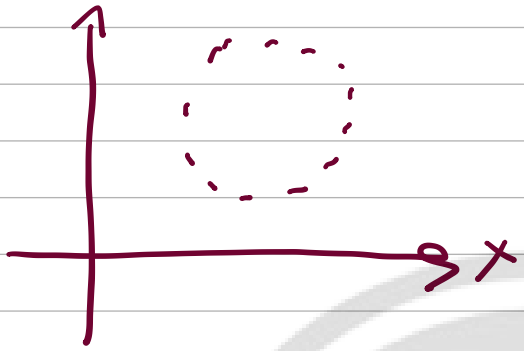


$0 < r < 1$ (No straight line)

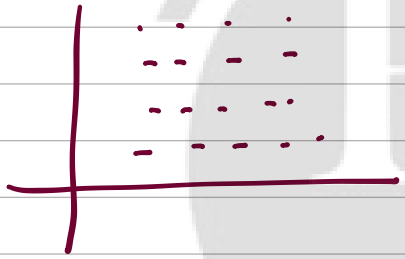


$-1 < \gamma < 0$

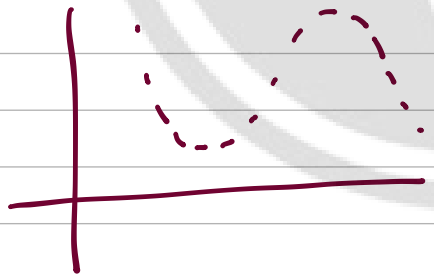
No straight Line



$\gamma = 0$



$\gamma = 0$



$\gamma = 0$

→ Exact magnitude can not be calculated in scattered Diagram method

Karl Pearson's coefficient correlation

$$r = \frac{cov.(x,y)}{\sigma_x \sigma_y}$$

or

$$r = \frac{\Sigma (x_i - \bar{x}).(y_i - \bar{y})}{N \sigma_x \sigma_y}$$

$$r = \frac{\Sigma (x_i - \bar{x}).(y_i - \bar{y})}{\sqrt{\Sigma (x_i - \bar{x})^2 \times \Sigma (y_i - \bar{y})^2}}$$

or

$$r = \frac{\Sigma xy - \frac{\Sigma x \times \Sigma y}{N}}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{N}} \sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{N}}}$$

→ covariance $= cov(x,y) = \frac{\Sigma (x_i - \bar{x}).(y_i - \bar{y})}{N}$

→ $cov(x,y)$ can be any Real number (Negative, Positive & zero)

→ $cov(x,y)$ does not change with origin but it changes with change of scale

→ Correlation does not change with change of origin

i.e. $u_i = x_i - A$ & $v_i = y_i - B$

then

$$r = \frac{\Sigma uv - \frac{\Sigma u \times \Sigma v}{N}}{\sqrt{\Sigma u^2 - \frac{(\Sigma u)^2}{N}}\sqrt{\Sigma v^2 - \frac{(\Sigma v)^2}{N}}}$$

→ Correlation does not change with change of scale (provided scale is positive)

→ when scale is negative, the sign of correlation may change but magnitude does not change

e.g. $r(x,y) = 0.5$

If $u_i = 2x_i$ & $v_i = 4y_i$

then $r(u_i, v_i) = (+)(+)\,0.5 = 0.5$

e.g. $r(x,y) = 0.6$

If $u_i = -2x_i$ & $v_i = 3y_i$

then $r(u,v) = (-)(+)\,0.6 = -0.6$

e.g. If $r(x,y) = 0.7$

& $u_i = -2x_i$ & $v_i = -4y_i$

then $r(u,v) = (-)(-)\,0.7 = (+)\,0.7$

Spearman's Rank correlation

→ This method used for qualitative characters & level of agreements & disagreements b/w opinions of Judges

→ when no numbers repeat

$$r = 1 - \frac{6 \Sigma D^2}{N^3 - N}$$

when some numbers repeat

$$\Rightarrow \quad r = 1 - \frac{6\left[\Sigma D^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2)\right]}{N^3 - N}$$

→ $\Sigma D = 0$

X	Y	R_1	R_2	$D = R_1 - R_2$	D^2
				0	

Concurrent Deviation method

Concurrent Deviation
⇓
when x & y both increase or both decrease

$$r = \pm\sqrt{\pm\left(\frac{2c-m}{m}\right)}$$

where c = Total no of concurrent Deviations
$m = n-1$

	x	y		Concurrent Deviation
+	10	12	+	yes (+)
+	12	15	-	No (-)
-	15	14	+	No (-)
+	14	15	+	yes (+)
-	16	18	+	No (-)
	12	20		

$c = 2$

$m = 6-1 = 5$

$$r = \sqrt{\frac{2(2)-5}{5}}$$

$$r = \sqrt{\frac{-1}{5}}$$

$$r = -\sqrt{\frac{1}{5}}$$

→ when $\frac{2c-m}{m}$ is negative then (-) sign will be taken out from $\sqrt{}$ sign

Bivariate Frequency Distribution Table

maths \ stats	0-5	5-10	10-15	15-20	Total
0-10	2	5	1	0	8
10-20	3	2	3	4	12
20-30	4	1	2	6	13
Total	9	8	6	10	33

Frequency Distribution of marks of maths

Marks	No of Students
0-10	8
10-20	12
20-30	13
	33

This is called Maginal Distribution

Frequency Distribution of marks in maths when score in stats is '5-10'

Marks	No of Students
0-10	5
10-20	2
20-30	1
	8

This is called Conditional Distribution

For Bivariate Distribution table of "m x n"

Total cells = m x n

Total marginal Distribution = 2

Total conditional Distribution = m + n

Coefficient of Determination

$$= r^2 = \frac{\text{Explained variance}}{\text{Total variance}}$$

Coefficient of Non Determination

$$= 1 - r^2$$

Regressions

→ Establishing mathematical relation b/w two variables (Independent & Dependent)

→ Prediction of Dependent variable

→ for Linear Regression Least Square method is used

There two Linear Regression Lines

	when y depends on x	when x depends on y
Standard form	$y = a + bx$	$x = a + by$
How to find = ?	use formula $y - \bar{y} = b_{yx}(x - \bar{x})$	use formula $x - \bar{x} = b_{xy}(y - \bar{y})$
slope of line	slope = b_{yx}	slope = $1/b_{xy}$

b_{yx} & b_{xy} are know as Regression coefficients

Calculation of Regression coefficients

$$b_{yx} = \frac{cov(x,y)}{\sigma_x^2}$$

$$b_{yx} = \frac{\Sigma(x_i - \bar{x})\cdot(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$$

$$b_{yx} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{N}}{\Sigma x^2 - \frac{(\Sigma x)^2}{N}}$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x}$$

$$b_{xy} = \frac{cov(x,y)}{\sigma_y^2}$$

$$b_{xy} = \frac{\Sigma(x_i - \bar{x})\cdot(y_i - \bar{y})}{\Sigma(y_i - \bar{y})^2}$$

$$b_{xy} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{N}}{\Sigma y^2 - \frac{(\Sigma y)^2}{N}}$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y}$$

$r = \pm\sqrt{b_{yx} \times b_{xy}}$

→ 'r' will be positive if both b_{yx} & b_{xy} are positive

→ 'r' will be negative if both b_{yx} & b_{xy} are negative.

→ $b_{yx} \times b_{xy} \leq 1$

$r \leq \frac{b_{yx} + b_{xy}}{2}$

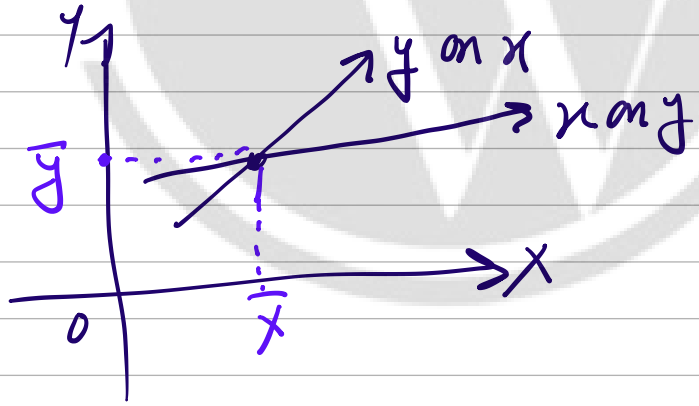
If Regression Line y on x is: $ax+by+c=0$

then $b_{yx} = -\frac{a}{b}$

If Regression Line x on y is $Ax+By+C=0$

then $b_{xy} = -\frac{B}{A}$

Two Regression lines intersect each other at $(\bar{x}, \bar{y})$



Regression coefficients does not change with change of origin

$u_i = x_i - A$ & $v_i = y_i - B$

then $b_{yx} = b_{uv}$ & $b_{xy} = b_{uv}$

Regression coefficients changes with change of scale

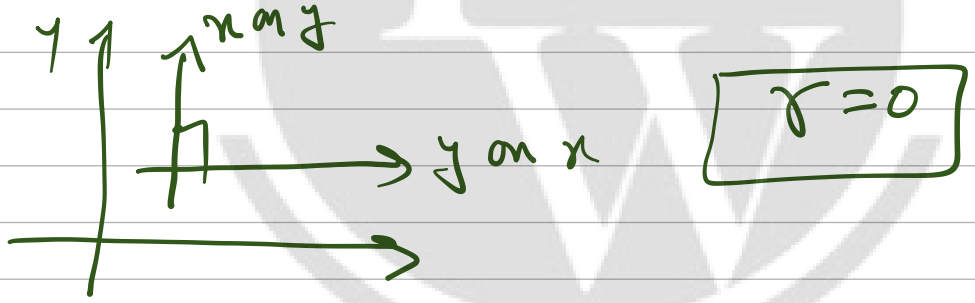
$u_i = a x_i + b$ & $v_i = c y_i + d$

So $b_{vu} = \frac{c}{a} \times b_{yx}$

or

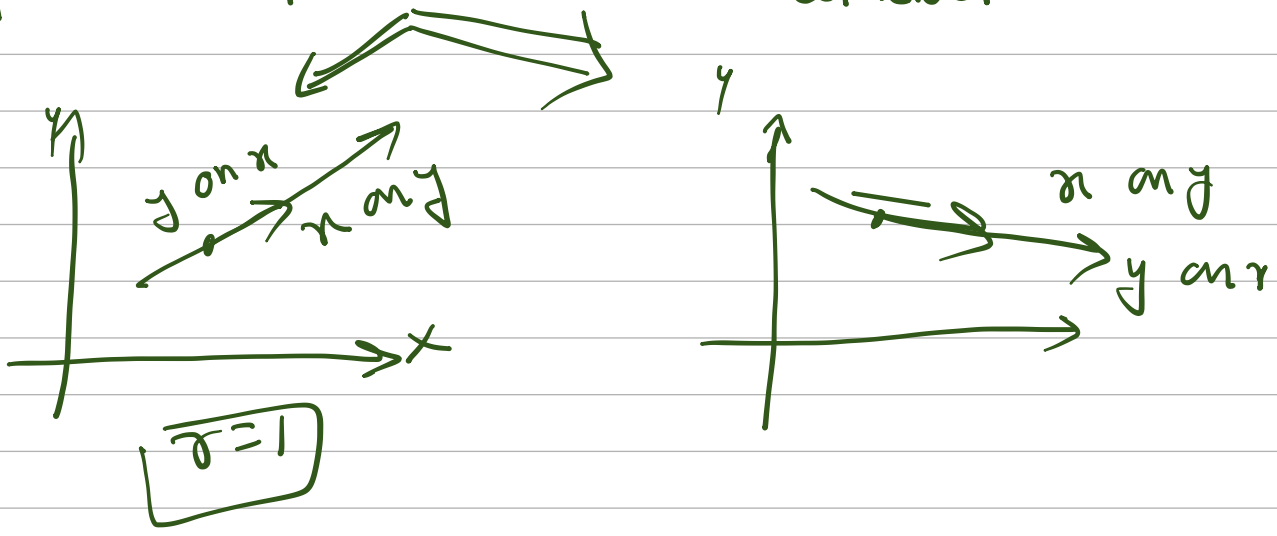
$b_{vu} = \frac{\text{scale of } y}{\text{scale of } x} \times b_{yx}$

when two lines are perpendicular



$r = 0$

when two lines are coincident



$r = 1$